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Journal of Sound and Vibration 266 (2003) 699–709
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Parameter estimation and multivariable model building for the non-destructive, on-line determination of eggshell strength

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Received 13 January 2003

Abstract

In the non-destructive quality assessment of agro-products using vibration analysis, the resonant frequency and the damping of the vibration are the main interest. Those parameters are usually calculated starting from the frequency spectrum, obtained after a fast Fourier transformation (FFT) of the time signal. However, this method faces several drawbacks when applied to short-time signals, as in the case of impact testing of highly damped specimen. An alternative to the FFT method is used for the high-resolution estimation of both resonant frequency and damping. Furthermore, the mass–spring model that is used in the literature for non-destructive quality assessment of various agro-products is extended with the incorporation of the damping and a shape characteristic. As a practical example, eggshell stiffness was estimated using vibration measurements. A data set consisting of 229 eggs was measured. It is shown that both the damping and the shape characteristics are of major importance to explain eggshell strength. This paper makes clear that a univariable model, as is mostly used in the literature, is not always satisfactory to describe the vibration behaviour of biological products.

1. Introduction

Due to higher speeds in commercial egg grading machines (which grade up to 120 000 eggs/h), an automated quality sorting principle is of interest to assure a consistent egg quality. One of the main physical quality parameters for consumption eggs is the presence of a crack in the eggshell. Very recent research shows that it is possible to detect cracks in eggshells on-line using vibration analysis [1–4]. For this purpose, the egg is hit four times around its equator and the similarity (correlation) between the four measurements is used as a sorting criterion. In this way, up to 90%

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0022-460X/03/$ - see front matter
doi:10.1016/S0022-460X(03)00595-9

of the cracks can be removed while the number of false rejects (the percentage of intact eggs that are classified as broken) remains well below 0.5% [1].

The research in this paper focuses on the question of whether it is possible to use vibration measurements to assess eggshell strength, as another important quality parameter, in an on-line way. This is a further step towards an integrated, on-line egg quality assessment.

In the literature, different techniques can be found for determining eggshell strength. In general, they can be split up into direct methods and indirect methods [5]. Indirect methods measure a parameter that is related to the eggshell strength. The correlation between the different methods is moderate, and the choice of which method to choose often depends on the application. Most methods are destructive.

Measuring the eggshell thickness is one of the frequently used indirect methods to have an indication of the eggshell strength. Using a micrometer, it is possible to determine the thickness up to 0.02 mm. Another indirect measure for the strength of the egg is provided by the calculation of the percentage eggshell. Abdallah et al. [6] show that 80% of the percentage breakage of a batch of consumption eggs can be explained by this percentage. A third widely used indirect method makes use of the quasi-static compression of the eggs between two parallel plates. By measuring the force deformation curve, it is possible to determine the static stiffness k_{stat} of the egg. In the present study, this method is used as a reference.

Different direct methods are described in the literature. The most widely used is the compression fraction force measured during quasi-static compression [6,7]. Other methods include puncture tests and impact tests. All direct methods are destructive.

An alternative to those tests is provided by vibration analysis. The backbone of this research is formed by the experimental and theoretical modal analysis of the product. In case of spherical objects, this modal shape is an elliptical deformation, as has been observed both by theoretical and practical modal analysis [3]. An example of an experimental modal analysis is given in Fig. 1, where the first elliptical mode shape of a chicken egg is shown.

The information of the vibration behaviour is used in the optimization of the test set-up. The set-up must be chosen in such a way that the vibration of the object is influenced as minimally as possible. From the literature, it appears that their behaviour has been simulated as a mass–spring system from which the stiffness of the product is the factor describing its quality. The stiffness of the product is hence a function of both the mass of the object m and its resonant frequency given by [3]:

$$k_{dyn} \sim m \times RF^2, \tag{1}$$

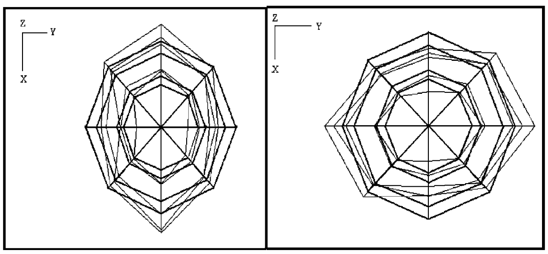

Fig. 1. Experimental modal analysis of a chicken egg [5]. Dark line: object in rest; bright line: elliptical displacement.

with k_{dyn} the dynamic stiffness, m the mass of the egg and RF the resonant frequency. This clearly is a univariable model, linking only one vibration parameter (the dynamic stiffness k_{dyn}) with a reference quality parameter.

Coucke [3] used the dynamic stiffness to estimate the static stiffness k_{stat} of the eggshell. This univariable model is not applicable as such for eggshell strength assessment because of the moderate correlation between the k_{dyn} and k_{stat} [8]. The current study focuses on the expansion of this univariable model to obtain more accurate estimates of the eggshell strength. Additional and improved information will be provided by:

(1) a very accurate estimation of the resonant frequency (note that it is quadratically related to the dynamic stiffness and hence plays a crucial role);
(2) expanding the model by incorporating the damping of the vibration, which was ignored in all research found in the literature;
(3) including the egg shape, since it determines how forces applied to the egg are distributed over the eggshell.

2. Materials and methods

2.1. Estimation of vibration parameters

Classically, the resonant frequency and damping corresponding to a certain mode shape are estimated using the frequency spectrum [3,8]. However, research has shown that obtaining the frequency spectrum using the fast Fourier transform (FFT) does not result in very accurate estimators in case of short-time signals [9]. In general, constructing the frequency spectrum using an FFT can be regarded as a way to reconstruct the power distribution of a time signal as a function of the frequency. As mentioned before, the parameters of interest are the resonant frequency and the corresponding damping of the vibration, rather than the whole energy distribution over 'all' frequencies up to the Nyquist frequency ('spectral line estimation'). Starting from the frequency spectrum, the resonant frequency is calculated as the frequency with the largest power. The frequency resolution of the frequency spectrum is set by the ratio between the sample frequency and the number of points in the time signal. It is clear from this relation that highly damped specimens give rise to a low-frequency resolution. Altering the sample frequency has no effect on the resolution, since it also alters the number of points in the time domain. Since an impact is used as excitation method, there is no way to increase the number of *relevant* points in the time signal: the length is a characteristic of the specimen under study. So, the only way to enlarge the number of points without changing the energy content of the time signal is to adopt zero padding. Indeed, in this way the denominator of the equation becomes larger, giving a better resolution. For a discussion, see further in Ref. [9].

The corresponding damping of a given mode shape is calculated using the 'half-power width' [10]. In this method, the damping δ is often calculated using the formula [3]:

$$\delta = \frac{(w_2 - w_1)}{2w_n}, \tag{2}$$

where w_n denotes the resonant frequency, w_1 and w_2 the frequencies at which the power is half the power of the resonant frequency, at both sides of the resonant peak in the power spectrum. The damping factor calculated as such is often referred to as the damping ratio of the vibration. The error that is made by the estimation of the damping using the FFT is made up of three parts: (1) the error on the estimation of the vibration power at each of the three frequency lines (due to, for instance, environmental noise), (2) by the interpolation that is needed in sparse spectra to find the frequencies w_1 and w_2 and (3) the finite contact time between impactor and egg so that the impact spectrum is not white.

Due to those drawbacks of the FFT, another method was used to find robust and high-resolution estimators of the resonant frequency and damping. High-resolution estimators, which are even called 'super resolution estimators' [9], distinguish themselves from low-resolution estimators (such as the FFT) by the fact that their aim is not to construct the power distribution of a given time signal over all frequencies. Instead, their aim is to find precise knowledge about one line in a given spectrum ('spectral line estimation'). The term high resolution is justified since they are able to resolve spectral lines separated in frequency f by less than $1/N$ cycles per sampling interval, which is the resolution limit for classical periodogram-based methods [9].

Furthermore, an important difference between the periodogram-based (low-resolution) estimators and the high-resolution estimators is the fact that the periodogram-based methods do not assume a 'parametric' form of the data. On the other hand, the high-resolution estimators exploit an exact parametric description of the signal (in this case a damped cosine). This stresses also the weak point of the high-resolution estimators, namely one has to assume this parametric form to obtain good estimators—a wrong parametric form may lead to wrong parameter estimates. In the impulse–response measurements taken for the non-destructive quality assessment, this parametric form is easily derived by solving the equation of motion for a mass–spring–damper system, which leads to the parametric form of a damped cosine. In this study the simplification towards a single degree-of-freedom system is made, since the set-up is optimized in such a way that only the first modal shape is encountered in the response. This knowledge follows directly from the experimental modal analysis of the products [3]. Denoting the noise term by z, the acquired signal y is the sum of the parametric form s and the noise z:

$$y_n = s_n + z_n \quad \text{for } n = 1, \ldots, N, \tag{3}$$

where N denotes the number of points acquired. An intuitively appealing approach to spectral line estimation consists of determining the unknown parameters as the minimizers of the following criterion:

$$F = \sum_{n=1}^{N} (y_n - \hat{y}_n)^2 = \sum_{n=1}^{N} [y_n - (A\mathrm{e}^{-\delta n} \cos(wn + \varphi)]^2, \tag{4}$$

where w denotes the frequency, A the amplitude, φ the phase and δ the damping. Since F is a non-linear function of the four arguments, the method that obtains parameter estimates by minimizing Eq. (4) is called the non-linear method of least squares. The solution of this equation is given by

$$\Gamma^{-1} = 2\sum_{n=1}^{N} c_n c_n^{\mathrm{T}} \exp(2c_n^{\mathrm{T}}\ \mathrm{Re}(\hat{a}_0)),$$

$$\hat{a} = \hat{a}_0 + 2\mu\Gamma \sum_{n=1}^{N} c_r\ \exp(c_n^{\mathrm{T}} a_0^{*})(y_n - \exp(c_n^{\mathrm{T}}\hat{a}_0)). \tag{5}$$

The parameter vector $\hat{a}$ contains the four parameters that describe the parametric form of the damped cosine. Of interest here are only the damping δ and the resonant frequency RF. A more in-depth study about this type of minimization is provided by De Ketelaere and De Baerdemaeker [11] and Golden and Friedlander [12].

In the analysis that follows, both high- and low-resolution (using the FFT) estimators for the resonant frequency and damping are compared.

2.2. Measurement set-up

A desktop unit was constructed in order to test automatically a large number of eggs (229 eggs were measured). The set-up is shown in Fig. 2 and consists of three major parts:

(1) The support of the egg is chosen in such way that they coincide with the nodal points of the first elliptical (also called S 20) modal shape (see Fig. 3). On the other hand, the triangular (S 30) mode will be suppressed artificially. The supports consist of hard rubber diabolo-shaped rollers. The egg spins around its long axis.
(2) A small impact hammer is used as an excitation device. It consists of a nylon rod with a small plastic ball glued on it.
(3) The vibration of the egg is captured by a small MONACOR® microphone (electret tie microphone, type 2005).

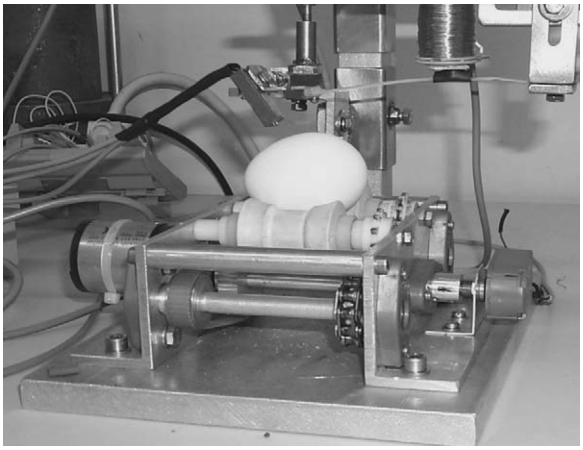

Fig. 2. The desktop unit with diabolo-shaped rollers, microphone and impact hammer.

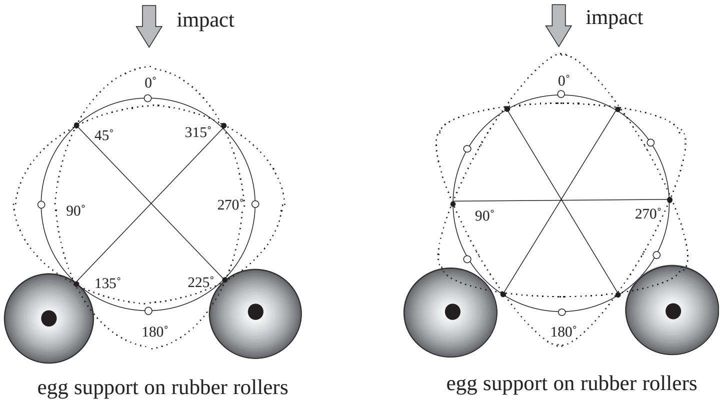

Fig. 3. The support of the egg is placed at the nodal points of the elliptical S 20 mode (left). The triangular mode (S 30) will be damped artificially by the support (right). Figure taken from Coucke [3].

A software program was written in LabVIEW® that allows a fast acquisition and processing of the vibration data. A National Instruments (National Instruments, USA) 12-bit data-acquisition board (AT-MIO 16E2) was used for the acquisition of the data, at a sample rate of 50 kHz. The resonant frequency of the elliptical modal shape of an egg is situated between 3 and 6 kHz. Four measurements are performed around the equator of the egg at equidistant points while spinning. The parameters that are calculated in the vibration experiment are the following:

(1) The resonant frequency of the egg, both using the low- and the high-resolution estimators. For the low-resolution estimation, the highest peak in the FFT is taken as resonant frequency.
(2) The damping, both using the low- and the high-resolution estimators. For the low-resolution estimation, the half-power width is used, as explained in a previous paragraph with Eq. (2).
(3) The dynamic stiffness in Eq. (1) using the low- and high-resolution estimate for the resonant frequency.
(4) The mass of the egg, with an accuracy of 0.1 g.
(5) The length and the width of the egg, measured with sliding callipers, with an accuracy of 1 mm.

The static stiffness served as reference measurement and was measured at four equidistant places at the equator of the egg using the Universal Test Machine (Universele Test Systeme GmBh, Germany), shown in Fig. 4. The compression speed was set to 1 mm/min. A maximal force of 10 N was exerted. The slope of the force–deformation curve was calculated using linear regression in the range 0.98–10 N. This slope is a measure for the static stiffness k_{stat}. Typical values for the static stiffness of an egg ranged between 80 and 200 kN/m. Differences of 10 kN/m were measured between the highest and lowest static stiffness measured on an individual egg due to the non-homogeneous eggshell distribution around the equator.

2.3. Data analysis

The data were processed with the statistical software package SAS (SAS version 6.12, the SAS Institute Inc., USA). A biplot based on the two first principal components [13,14] was constructed

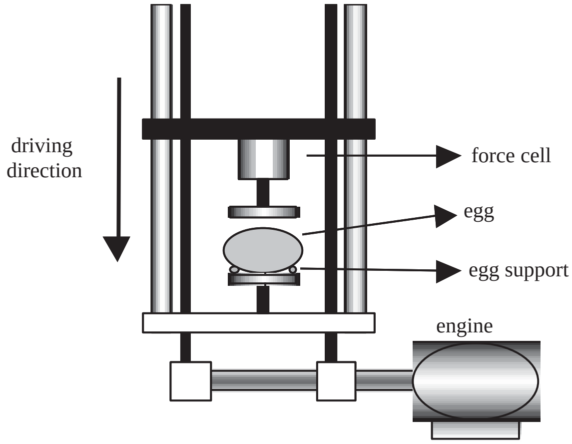

Fig. 4. Schematic representation of the Universal Test Machine. Figure taken from [3].

in order to have a first global overview of the multi-dimensional data (there are nine variables). For the principal component analysis, the correlation matrix was used. The stars on the plot give the projected observations on the 2D plane, while the arrows indicate the influence of the parameters on the observations. The angle between two arrows (variables) is an indication for the relation between them. An angle of 0° (180°) indicates that the two variables show a correlation of 1 (-1). An angle of 90° or 270° denotes no correlation.[1]

An outlier detection procedure was performed. Several criteria are described in the literature for this purpose and were used in this research. Methods used are the leverage values of the hat matrix H, the deleted studentized residuals t_i, DFFITS and the Cook's distance. See Neter et al. [15] for an extended explanation on this topic.

A multiple linear regression model was built. In order to find the parameters that are relevant in the description of k_{stat}, the STEPWISE procedure of the PROC REG statement in SAS was used to select the most relevant parameters [15].

3. Results and discussion

3.1. Data representation

Using the first two principal components for the biplot construction, 72.8% of the variation in the data set was explained. This is shown in Fig. 5. The resonant frequencies calculated with the low- and the high-resolution methods overlay each other in Fig. 5 ($r = 0.99$, $P < 0.0001$). Hence, only one label is given for the two arrows. The same holds for the dynamic stiffness. For the damping calculated by the two methods, the correlation is lower ($r = 0.86$, $P < 0.0001$), indicating that they might give some different information.

[1] Care has to be taken while interpreting the relation between two variables by the angle between their arrows. Also the length of the arrows has to be taken into account. A short arrow indicates that not all of the variation in that parameter is explained in the two-dimensional (2D) plot. The maximal length of an arrow is 1.

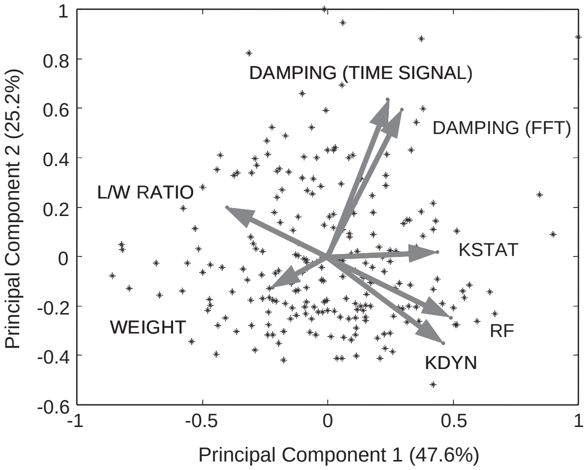

Fig. 5. Biplot of the data of all 229 eggs. Stars indicate 2D projections of each egg data. Arrows indicate variable orientation, and the angles between them can be interpreted as correlations since principal component analysis was based on the correlation matrix.

The resonant frequencies show up to be highly related to the dynamic stiffness k_{dyn} ($r = 0.87$, $P < 0.0001$). This was to be expected, since the resonant frequency squared is related to the dynamic stiffness in Eq. (1). The mass plays no significant role ($r = 0.05$, $P = 0.44$) proving that larger eggs are not weaker than smaller eggs in terms of k_{dyn}.

A negative correlation was found between the static stiffness and the shape index L/W ($r = -0.33$, $P < 0.0001$). This relation was also found by Essary et al. [16], who remarked that the shape of the egg explained a major part of eggshell cracks, and hence gives an indication about its strength. This relation was even stronger for the dynamic stiffness ($r = -0.71$, $P < 0.0001$).

The correlation between the static and dynamic stiffness was 0.72 ($P < 0.001$), indicating that 53% of the variance of the static stiffness is explained by the dynamic stiffness. In an attempt to describe the remaining 47% of the variance, a multivariable model was built.

3.2. Outlier detection

Using the four above-mentioned criteria for outlier detection, an observation is considered to be an outlier if at least two criteria indicate it so. A summary of the outlier analysis is given in Table 1. Five outlying observations are omitted from the data set in the further analysis.

3.3. Multivariable model building

For the resulting 224 eggs, a multiple linear regression model was set up. The covariates are the resonant frequency and the damping, calculated with both low- and high-resolution estimators, the dynamic stiffness (containing also the mass) and the shape index of the egg. The static stiffness serves as an output variable.

Table 1
Results of the outlier research

Egg no.	h_{ii}	t_i	$DFFITS_i$	D_i
20	*	*	*	*
80	*		*	
136	*		*	
147	*		*	
149	*	*	*	*

* indicates an outlying observation for a given criterion. Observations are treated as outliers if at least two criteria indicate so.

Table 2
Stepwise model building procedure for the determination of the static stiffness k_{stat}

Variable		R_p^{2a}	F	$Prob > F$
In	Out			
k_{dyn}	—	0.5275	247.88	0.0001
δ_t[b]	—	0.6885	114.18	0.0001
L/W	—	0.8069	134.87	0.0001

[a] R_p^2: coefficient of determination of the model with p parameters.
[b] δ_t: the high-resolution damping estimator.

Applied to the data set, the k_{dyn} is the most important parameter (it is entered as the first variable in the model). It explains about 53% of the variation of the static stiffness (see the R^2 value in Table 2). This agrees with a prior study conducted by Coucke et al. [8], who modelled the egg as a mass–spring system and reported a similar result. The second most important parameter was the damping δ_t, calculated using the high-resolution estimator. With both k_{dyn} and δ_t in the model, about 69% of the variation of the k_{stat} can be explained, a considerable improvement over the univariable model. This result clearly proves the importance of the high-resolution estimators, and also the use of a multivariable model to describe the physical eggshell quality. In a third step, the shape index L/W is added to the model, raising the explained variance to above 80%. The coefficient of correlation r, which is the square root of the coefficient of determination R^2 given in Table 2, between the predicted static stiffness and the measured static stiffness, hence becomes 0.90. No other variables met the entry rule.

It was notable that the damping calculated as the half-power width gives no statistical relevant information about the static stiffness given the other parameters. The resonant frequencies themselves do not appear in the model. This fact is caused by the high correlation between the resonant frequency and the dynamic stiffness, as could already be seen from the biplot (Fig. 5).

The final model for the prediction of the static stiffness is given by

$$k_{stat} = -244748.6 + 9.66k_{dyn} + 20095.46\delta_t + 1588.08L/W. \quad (6)$$

With this relation, it is possible to estimate the static stiffness of an egg, which is still a wide-used quality parameter, using vibration analysis. The main advantage is the quick nature of the measurement: with the prototype given in Fig. 2, it is possible to assess the static stiffness of up to 1000 eggs within one hour, compared to around 25 eggs in the classical way using a universal test machine.

The question that arises here is whether one should still hold onto the static stiffness as the reference quality parameter describing eggshell strength, or use the dynamic stiffness as such as a new parameter. One reason for preferring the dynamic stiffness above the static stiffness is that in practice, dynamic forces rather than static forces cause eggshell breakage, for instance when an egg rolls and hits the battery cage. However, if one would like to incorporate the dynamic stiffness as a selection parameter for new strains, new research is needed to determine its heritability.

4. Conclusions

A multivariable model including high-resolution estimators of the vibration characteristics has been built in a practical example of eggshell strength assessment in a non-destructive way. The univariable model found in the literature that relates the dynamic stiffness directly to the static stiffness has been broadened by the incorporation of a high-resolution estimation of the vibration damping and a parameter describing the shape of the object. Using this multivariable model, the coefficient of correlation between the k_{stat} and the prediction using higher-mentioned parameters, increases from about 0.7 to 0.9. This makes it possible to obtain a quick (up to 1000 eggs/h) estimate of the static stiffness of an egg using vibration analysis.

Acknowledgements

The authors gratefully acknowledge the IWT–Vlaanderen (Instituut voor de Bevordering van het Wetenschappelijk Technologisch Onderzoek Vlaanderen, Doctoral Grant No. 981283) and the FWO (Fonds voor Wetenschappelijk Onderzoek in Vlaanderen) for the financial support and MOBA BV (Barneveld, The Netherlands) for their technical support.

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